Mathematics and tins

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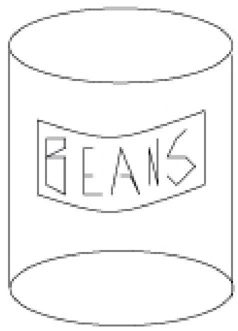
20 juin 2006

The quote of the week

"The calculus is the greatest aid we have to the application of physical truth in the broadest sense of the word"
Osgood, W. F. in *Mathematical Maxims and Minims*, Raleigh NC :Rome Press Inc., 1988

Tins

Remember the previous problem I asked you to solve :
A tin is cylindrical and its volume must be one liter $1000cm^3 = 1dm^3$



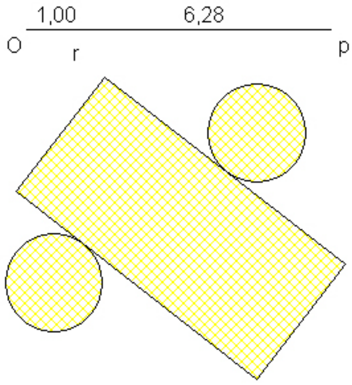
What will be the dimensions of this box so that the metal area would be minimum ?

Solution :

The volume of a cylindar of height h and of radius R is given by the formula :

$$V = \pi R^2 h \quad (1)$$

In order to calculate this area we can developp the box as shown on this figure :



let R be the radius and h be the height.

The metal area which is necessary to construct this box is the area of the two circles (the lids of the box) and the area of the rectangle. So, we find, using equation (1) :

$$\begin{aligned} A &= 2\pi R^2 + 2\pi R \times h && (2) \\ &= 2\pi R^2 + 2\pi R \times \frac{1000}{\pi R^2} && (3) \\ &= 2\pi R^2 + 2 \times \frac{1000}{R} && (4) \end{aligned}$$

A is a function of the radius R. Hence, the domain of this function is R^+, and we can find the minimum of this function using the derivative function :

$$A'(R) = \frac{4(\pi R^3 - 500)}{R^2}$$

The minimum of the function is reached for the value of R so that $A'(R) = 0$, hence :

$$R = \sqrt[3]{\frac{500}{\pi}}$$

In these conditions, the height h of the box is :

$$h = \frac{1000}{\pi \left(\sqrt[3]{\frac{500}{\pi}} \right)^2}$$

The work of the week

1. Read, understand and explain the previous calculations.
2. Prove that $h = 2R$ and verify this calculation on a real can !
3. Do the same work for a $200cm^3$ can.
4. Draw the curve of the areas related to the radius R in the two previous cases.
5. Present this results on a poster !